A Precise Integration Method in Time Domain for Switching Transient Analysis on Long Nonuniform Transmission Lines

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***Abstract* — This paper uses precise integration method in time domain (PITD) to simulate the electromagnetic transient behavior of the long nonuniform transmission line. This method does not have the Courant-Friedrichs-Lewyc (CFL) condition restrict which restrict the time step of FDTD method. It means in PITD method large time steps can be adopted to achieve accurate results efficiently. The main computation cont of this method is the calculation of the matrix exponential. In this paper, the matrix exponential integrator is computed using Krylov subspace method, and a scaling method is proposed to reduce the computation time of the matrix exponential.**

I. INTRODUCTION

In the calculation of transient behavior on transmission line during the switching transient, modeling of the transmission line is very important to accurate results. Many models proposed before are based on uniform transmission line assumption. However, if considering the sagging of the transmission line, the parameters varies along the line; the nonuniform lines (NULs) model would be more accurate than uniform ones.

Many literatures used FDTD method to solve NULs, because it can conveniently takes the variation of per-unit-length parameters into account. However, conventional FDTD method is restricted by CFL criterion. For stability of the calculation, the time step of the FDTD method is bounded by the wave travel time on one spatial segment. When using FDTD method to deal with the switching transient of the NULs, in order to reflect the longitude variation of the transmission line, the spatial step is not allowed to be big (usually less than one hundred meters), then the time step should be less than one microsecond. Due to the long duration of the switching transient (typically more than hundreds of milliseconds), the simulation should be performed in millions of time steps. For lossless line, the FDTD recursive solution is very fast, this number of time steps seems to be acceptable, but considering the extra cost brought by the handling of frequency dependent impedance on every single spatial segment, the computation time might be too long. In literature [3], ADI-FDTD is developed to avoid the CFL criterion. As for ADI-FDTD method, although it is free of CFL criterion, the severe numerical dispersion and splitting error inhibit its application [3].

The PITD method [2], which is a semi-analytical method, neither has the CFL restrict nor is plagued by the severe numerical dispersion. However, the PITD method needs the calculation of the matrix exponential, which requires about $O(n^3)$ times of multiplication and $O(n^2)$ storage. The high computation cost prevents PITD's application in simulating long nonuniform lines. This paper uses Krylov subspace to approximate the matrix exponential integrator. In order to reduce the times of iterations in the Arnoldi process, a scaling technique is proposed to improve the condition of the matrix.

II. PRECISE INTEGRATION TIME DOMAIN SOLUTION OF LOSSY NONUNIFORM TRANSMISSION LINE

The Telegrapher's equation of single phase NULs is:

$$\frac{\partial}{\partial x}v(x,t)+R(x)i(x,t)+L(x)\frac{\partial}{\partial t}i(x,t)=0 \qquad (1)$$

$$\frac{\partial}{\partial x}i(x,t)+G(x)v(x,t)+C(x)\frac{\partial}{\partial t}v(x,t)=0 \qquad (2)$$

where *v(x,t)*and *i(x,t)* are the line-neutral voltage and the line current respectively, *R(x), L(x), G(x)* and *C(x)* are the per-unit-length parameters of NULs, respectively.

Spatially discretizing the partial differential equations (1) and (2) in a way the same as FDTD method gives:

$$\frac{d}{dt}i_n=-\frac{v_{n+1}-v_n}{\Delta x L_n}-\frac{R_n i_n}{L_n} \qquad (3)$$

$$\frac{d}{dt}v_{n+1}=-\frac{i_{n+1}-i_n}{\Delta x C_{n+1}}-\frac{G_{n+1}v_{n+1}}{C_{n+1}} \qquad (4)$$

Arrange the voltage and current in one vector as $X=[v_1,i_1,\ldots,i_m,v_{m+1}]^T$, the equations above can be given by:

$$\frac{d}{dt}X=AX+B \qquad (5)$$

where A is a tridiagonal matrix and $B=[2i_0/C_1,0,\ldots,-i_{m+1}/C_{m+1}]^T$ is the inhomogeneous term. In this problem, B is dependent on the boundary conditions. The solution of (5) can be written as:

$$X(t)=\exp(At)X(0)+\int_0^t \exp[A(t-\tau)]B(\tau)d\tau \qquad (6)$$

Discrete expression (6) in the time domain with the time step τ, and it can be solved in a recursive method:

$$X(t_{j+1})=\exp(A\tau)X(t)+\int_{t_j}^{t_{j+1}}\exp[A(t_{j+1}-\varsigma)]B(\varsigma)d\varsigma \qquad (7)$$

In order to evaluate the integral in expression (7), we can simplify the inhomogeneous term B as a step wave function within each time step:

$$B(t)=\begin{cases} r_0 & t_j < t < t_{j+1} \\ r_1 & t_{j+1} < t < t_{j+2} \end{cases} \quad (8)$$

where r_0 and r_1 are known vectors. As in switching transient, the frequency of is not very high, this assumption can be quite reasonable for practical application.

Substituting (8) into (7), the time-step integration expression can be transformed into the recursive form

$$X^{j+1} = \exp(A\tau)(X^{j} + A^{-1}r_0) - A^{-1}r_0 \quad (9)$$

When the initial value is given, the state variable X can be calculated in this recursive way.

III. EFFICIENT COMPUTATION OF THE MATRIX EXPONENTIAL OPERATOR

The main computation cost of evaluating the above formula comes from calculating the term of $e^{At}v$. Inspired by the solution of large scale linear equation system, in [1] a method based on Krylov space approximation is proposed to calculate $\exp(A\tau)v$ instead of the direct calculation. The basic idea of this method is to approximate $\exp(A\tau)v$ in Krylov subspace built by Arnoldi process.

Method of Krylov subspace for matrix exponential is first proposed and discussed is [4]. The basic idea of this method is to approximate matrix exponential with matrix polynomial taken from Krylov subspace.

$$K_m \equiv span\left\{v, Av, \cdots, A^{m-1}v\right\} \quad (10)$$

where m is the dimension of Krylov subspace, and K_m is an orthonormal basis of Krylov subspace which is built by Arnoldi process.

Aroldi process also produces an upper Hassenburg matrix H_m, and we have approximation of the matrix exponential

$$e^{A}v = \beta V_m e^{H_m} e_1 \quad (11)$$

where $(\beta = \|v\|)$.

For $\exp(A\tau)v$, the formula above can be written as:

$$e^{A\tau}v = \beta V_m e^{H_m \tau} e_1 \quad (12)$$

This immediately raises a question concerning the quality of this approximation. In [5], Saad proved the error of the approximation (12) is:

$$\left\| e^{A\tau}v - \beta V_m e^{H_m \tau} e_1 \right\|_2 \le 2\beta \frac{(\tau\rho)^m e^{\tau\rho}}{m!} \quad (13)$$

where $\rho = \|A\|$.

However, matrix A is not a well-conditioned matrix, because its elements, namely, 1/C and 1/L, differ from each other significantly. The bad condition of matrix A will make the computation of matrix exponential very inefficient.

In order to reduce the spectral radius of the coefficient matrix and accelerate the computation of the matrix exponential, this paper developed a scaling process to the matrix. The scaling scheme is:

1. Scale i_k to $i_k' = ai_k$, where a is 1/C, then the state variable becomes $X' = [v_1, ai_1, \ldots, ai_m, v_{m+1}]^T$ and the basic units of the matrix becomes $1/aC_k$ and a/L_k.
2. Solve the state equation $\frac{d}{dt}X' = A'X' + B'$
3. Calculate $i_k = i_k'/a$.

IV. CALCULATION EXAMPLE AND COMPARISON WITH FDTD METHOD

In the example, the method proposed above is used to study a typical switching transient problem, as shown in fig. 5. The input voltage source is a 1V sinusoidal wave of 50Hz with the initial phase 70°. The length of the transmission line is 240km and the distance between each two neighbor towers is 600m. The sagging of the line is 24m and the top of the line is 40m high from the ground.

The transient voltages at the near end of the transmission lines are depicted in Fig.1.

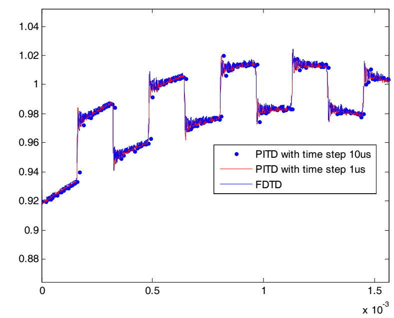

Fig.1. calculation results with FDTD and PITD method

From this example, we can get the conclusion that PITD method is enough accurate to solve switching transient of the transmission lines.

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